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Optimal observables for measuring three-gauge-boson couplings in $e^+e^- \to W^+W^-$

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Observables with optimised sensitivity to the WWZ and $WW\gamma$-couplings in $e^+e^- \to W^+W^-$ are investigated. They allow for separate studies of CP violation and of absorptive parts in the amplitude. We have calculated expected statistical errors on extracted couplings at $\sqrt{s} = 500$ GeV, with unpolarised or longitudinally polarised beams.

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1 Introduction

In this contribution we propose a method to measure the trilinear gauge couplings in the reaction $e^+e^- \to W^+W^-$. For the WWZ and $WW\gamma$ vertices we use the most general parametrisation consistent with Lorentz invariance of Hagiwara et al. [1]. It describes each vertex by seven form factors f_i, three of which correspond to CP violating couplings.

We shall consider the decay channels $W\,W \to \ell\,\nu_\ell +$ jet jet, where ℓ is a light lepton e or μ and the jets originate from a quark-antiquark pair. For these channels a complete kinematical reconstruction of the four-fermion final state is possible, apart from the ambiguity to associate the jets with the quark and the antiquark in case the jet charge cannot be measured.

In the following section we present a method to measure the form factors f_i with maximal statistical sensitivity, and in sec. 3 we describe some numerical results for the accuracy to be attained at a 500 GeV collider. Details can be found in [2].

2 The method of optimal observables

Let us denote by ϕ the full set of reconstructed kinematical variables, e.g. the scattering angle of the W^- in the c.m. frame and the decay angles of the W in their respective rest frames. Furthermore let g_i be the difference between f_i and its value in the SM at tree level. Since the trilinear gauge couplings enter linearly in the amplitude of our process the differential cross section can be written as

$$\frac{d\sigma}{d\phi} = S_0(\phi) + \sum_i S_{1,i}(\phi)\, g_i + \sum_{i,j} S_{2,ij}(\phi)\, g_i g_j \ , \tag{1}$$

where it is understood that one has symmetrised over any kinematical ambiguities. The idea of optimal observables is to measure the distribution of the functions[1]

$$\mathcal{O}_i(\phi) = \frac{S_{1,i}(\phi)}{S_0(\phi)} \tag{2}$$

and to determine the couplings from their mean values $\langle \mathcal{O}_i \rangle$. To first order[2] in the g_i one has

$$\langle \mathcal{O}_i \rangle = \langle \mathcal{O}_i \rangle_0 + \sum_j c_{ij}\, g_j \ , \tag{3}$$

[1]The functions $\mathcal{O}_i(\phi)$ defined here are available as a FORTRAN routine from the authors.

[2]A leading order expansion in g_i is used here, assuming that deviations from the SM at tree level are small, but this is not essential for the method to work.

from which the g_i can be extracted since $\langle\mathcal{O}_i\rangle_0$ and c_{ij} are calculable given (2) and (1). From the distribution of the $\mathcal{O}_i$ one also obtains the statistical errors on their mean values, and the observables $\mathcal{O}_i$ have been constructed to minimise the induced statistical errors on the extracted couplings g_i [3]. More precisely, it can be shown [2] that in the limit of small g_i they cannot be smaller in *any* other method, including a fit to the full distribution of the variables ϕ.

To have tractable expressions for the observables, some approximations such as taking the cross section (1) at tree level will be necessary in practice. They need however not be made in the extraction of the couplings: various theoretical and experimental effects such as radiative corrections or detector resolution might be taken into account when calculating the coefficients $\langle\mathcal{O}_i\rangle_0$ and c_{ij} in eq. (3).

This method is particularly well suited for testing discrete symmetries, because for an observable which corresponds to a CP violating coupling g_i, a nonzero mean $\langle\mathcal{O}_i\rangle$ is an unambiguous sign for CP violation in the reaction, provided detector and data selection are CP blind.[3] A similar statement holds for the study of absorptive parts in the scattering amplitude.

3 Numerical estimates for a 500 GeV collider

To estimate the statistical precision of our method we have calculated the errors in measuring real and imaginary parts of the form factors f_i^Z and f_i^γ of [1]. For an integrated luminosity of 10 fb^{-1} at $\sqrt{s} = 500$ GeV and unpolarised beams we find 1-σ errors between $2 \cdot 10^{-4}$ and $2 \cdot 10^{-2}$. With the additional information of the jet charge in each event they decrease by a factor between 1.2 and 2.7.

For some of the form factors the errors are strongly correlated. This holds in particular for pairs of f_i^Z and f_i^γ, because due to the interference of the WWZ and $WW\gamma$ vertices they appear as linear combinations in the amplitude. For left and right handed incident electrons these are, respectively,

$$\begin{aligned} f_i^L &= 4\sin^2\theta_W\, f_i^\gamma + (2 - 4\sin^2\theta_W)\, \frac{s}{s - M_Z^2}\, f_i^Z \\ f_i^R &= 4\sin^2\theta_W\, f_i^\gamma - 4\sin^2\theta_W\, \frac{s}{s - M_Z^2}\, f_i^Z\ , \end{aligned} \tag{4}$$

[3]Optimal observables have already been used to search for CP violation in τ-pair production at LEP1, with a clear gain of sensitivity over non optimised observables [4].

where θ_W is the weak mixing angle. The correlations for pairs f_i^L and f_i^R turn out to be much weaker. With the above parameters the errors are between $2 \cdot 10^{-4}$ and $2 \cdot 10^{-2}$ for the f_i^L, and between $5 \cdot 10^{-4}$ and $7 \cdot 10^{-2}$ for the f_i^R. With beams of left handed e^- or right handed e^+ one would exclusively measure the f_i^L. We find that given the same number of events the statistical errors on the f_i^L are almost the same with or without polarisation.

References

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